Geometry 16

24 August 2024 11:34

Q> $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots + \frac{1}{n\cdot \overline{n+1}\cdot \overline{n+2}} = \sum_{k=1}^{n} T_k$

$T_n = \frac{1}{n\cdot \overline{n+1}\cdot \overline{n+2}}$ $\quad V_n = \frac{1}{n\cdot \overline{n+1}}$

$V_n - V_{n+1} = \frac{1}{n\cdot \overline{n+1}} - \frac{1}{\overline{n+1}\cdot \overline{n+2}} = \frac{n+2-n}{n\cdot \overline{n+1}\,\overline{n+2}} = \frac{2}{n\cdot \overline{n+1}\cdot \overline{n+2}}$

$\Rightarrow \frac{V_n - V_{n+1}}{2} = T_n$

$\sum_{k=1}^{n} T_k = \frac{1}{2}\sum_{k=1}^{n}\left(V_k - V_{k+1}\right)$

Q> $\frac{1}{1\cdot4\cdot7} + \frac{1}{4\cdot7\cdot10} + \cdots + \frac{1}{\overline{3n-2}\cdot \overline{3n+1}\cdot \overline{3n+4}} = \sum_{k=1}^{n} T_k = \frac{1}{6}\sum_{k=1}^{n}\left(V_k - V_{k+1}\right)$

$T_k = \frac{1}{\overline{3k-2}\cdot \overline{3k+1}\cdot \overline{3k+4}}$ $\quad V_k = \frac{1}{\overline{3k-2}\;\overline{3k+1}}$

$T_k = \frac{V_k - V_{k+1}}{6}$

Q> $1^d + 2^d + 3^d + \cdots + n^d = S_n$

$d=0$ base case

$\downarrow$

$d=1 \to d=2 \to d=3$

$\downarrow$

$\vdots$ so on

$n^{d+1} - \overline{(n-1)}^{d+1} =$

$\vdots$

$2^{d+1} - 1^{d+1} =$

maximum power of d

Q> Using digits from 0 to 9 how many 6 digit numbers can be possible when 0, 2, 4 always sit together? (without repetition)

Ans:-

_ _ _ _ _ _

0 2 4 $\longrightarrow 4\times7\times6\times5$

_ 0 2 4 $\longrightarrow 6\times7\times6\times5$

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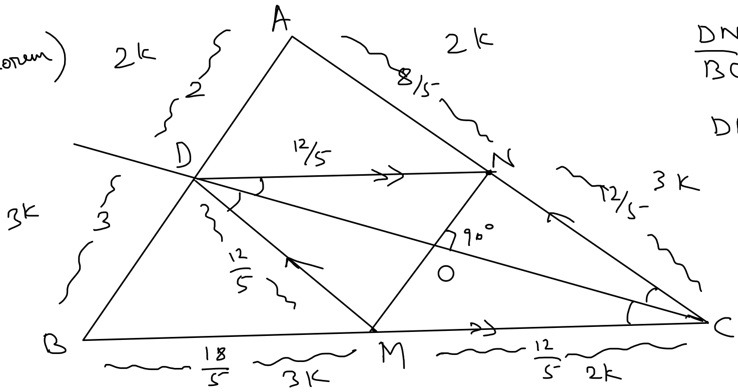
_ _ _ 0 2 4 $\longrightarrow 6\times7\times6\times5$

Q> Let ABC be a triangle with AB = 5, AC = 4 and BC = 6. The internal bisector of C intersects the side AB at D. Points M and N are taken on sides BC and AC such that DM || AC internal bisector of C intersects the side AB at D. Points M and N are taken on sides BC and AC such that DM||AC and DN||BC. If $MN^2 = \frac{p}{q}$ where p and q are relatively prime integers then what is the sum of the digits of $|p-q|$.



Ans:- $\frac{AD}{DB} = \frac{2}{3}$ (bisector theorem)

$AD = 2$
$BD = 3$

$\frac{DN}{BC} = \frac{2}{5}$

$DN = \frac{2}{5} \times 6$
$= \frac{12}{5}$

In ΔABC,

$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos C$

$\Rightarrow 5^2 = 4^2 + 6^2 - 2\times 4\times 6 \cos C$

$\Rightarrow 25 = 16 + 36 - 48 \cos C$

$\Rightarrow \cos C = \frac{27}{48} = \frac{9}{16}$

$|p-q| = 101$

In ΔMNC,

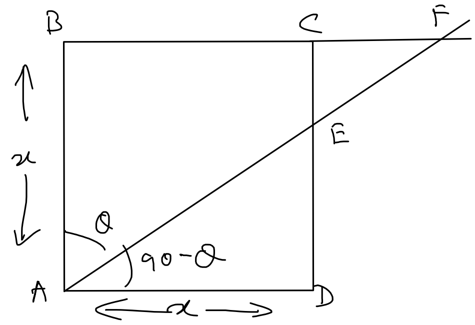
$MN^2 = NC^2 + MC^2 - 2(NC)(MC)\cos C$

$= \frac{144}{25} + \frac{144}{25} - 2\times\frac{144}{25}\times\left(\frac{9}{16}\right)$

$= 2\times\frac{144}{25}\left(1 - \frac{9}{16}\right)$

$= \cancel{2}\times\frac{\overset{18}{\cancel{144}}}{25}\times\frac{7}{\cancel{16}_{8}} = \frac{126}{25} = \frac{p}{q}$

Q) A straight line passing through the point A of a square ABCD intersects side CD at E and line BC at F. Prove that $\frac{1}{AE^2} + \frac{1}{AF^2} = \frac{1}{AB^2}$



Ans:- $AE \cos(90-\theta) = AD = AB$

$AF \cos\theta = AB$

$\Rightarrow AE = \frac{AB}{\sin\theta}$, $AF = \frac{AB}{\cos\theta}$

$\Rightarrow \frac{1}{AE^2} + \frac{1}{AF^2} = \frac{\sin^2\theta + \cos^2\theta}{AB^2} = \frac{1}{AB^2}$